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Electromagnetic Field Theory: A Problem Solving Approach

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SOLUTIONS TO SELECTED PROBLEMS

Chapter 1

1. Area = πa^2
3. (a) $\mathbf{A} + \mathbf{B} = 6\mathbf{i}_x - 2\mathbf{i}_y - 6\mathbf{i}_z$
 (b) $\mathbf{A} \cdot \mathbf{B} = 6$
 (c) $\mathbf{A} \times \mathbf{B} = -14\mathbf{i}_x + 12\mathbf{i}_y - 18\mathbf{i}_z$
5. (b) $\mathbf{B}_{\parallel} = 2(-\mathbf{i}_x + 2\mathbf{i}_y - \mathbf{i}_z)$, $\mathbf{B}_{\perp} = 5\mathbf{i}_x + \mathbf{i}_y - 3\mathbf{i}_z$
7. (a) $\mathbf{A} \cdot \mathbf{B} = -75$
 (b) $\mathbf{A} \times \mathbf{B} = -100\mathbf{i}_z$
 (c) $\theta = 126.87^\circ$
12. (a) $\nabla f = (az + 3bx^2y)\mathbf{i}_x + bx^3\mathbf{i}_y + ax\mathbf{i}_z$
14. (a) $\nabla \cdot \mathbf{A} = 3$
17. (b) $\Phi = \frac{1}{2}abc$
18. (a) $\nabla \times \mathbf{A} = (x - y^2)\mathbf{i}_x - y\mathbf{i}_y - x^2\mathbf{i}_z$
23. (b) $\nabla f = \frac{1}{h_u} \frac{\partial f}{\partial u} \mathbf{i}_u + \frac{1}{h_v} \frac{\partial f}{\partial v} \mathbf{i}_v + \frac{1}{h_w} \frac{\partial f}{\partial w} \mathbf{i}_w$
 (c) $dV = h_u h_v h_w du dv dw$
 (d) $\nabla \cdot \mathbf{A} = \frac{1}{h_u h_v h_w} \left[\frac{\partial}{\partial u} (h_v h_w A_u) + \frac{\partial}{\partial v} (h_u h_w A_v) + \frac{\partial}{\partial w} (h_u h_v A_w) \right]$
 $(\nabla \times \mathbf{A})_u = \frac{1}{h_v h_w} \left[\frac{\partial (h_w A_w)}{\partial v} - \frac{\partial (h_v A_v)}{\partial w} \right]$
25. (a) $r_{QP} = \sqrt{30}$, (b) $\mathbf{i}_{QP} = \frac{\mathbf{r}_{QP}}{r_{QP}} = \frac{\mathbf{i}_x - 5\mathbf{i}_y + 2\mathbf{i}_z}{\sqrt{30}}$,
 (c) $\mathbf{n} = \frac{5\mathbf{i}_x + \mathbf{i}_y}{\sqrt{26}}$

Chapter 2

3. $E_0 = \frac{4}{3} \frac{\pi R^3 \rho_m g}{q}$
4. $Q_2 = \frac{2\pi\epsilon_0 d^3 Mg}{Q_1 \sqrt{l^2 - \left(\frac{d}{2}\right)^2}}$
5. (a) $\omega = \left[\frac{Q_1 Q_2}{4\pi\epsilon_0 R^3 m} \right]^{1/2}$

$$7. (a) m = \frac{m_1 m_2}{m_1 + m_2},$$

$$(b) v = \pm \sqrt{\frac{-q_1 q_2}{2\pi\epsilon_0 m} \left(\frac{1}{r} - \frac{1}{r_0} \right)},$$

$$(d) t = \frac{\pi}{2} r_0^{3/2} \left[\frac{2\pi m \epsilon_0}{-q_1 q_2} \right]^{1/2}$$

$$8. h = \frac{qE_0 L^2}{2mv_0^2}$$

$$10. (b) q = \frac{6\sqrt{3}}{7^{3/2}} Q$$

$$12. (a) q = 2\lambda_0 a, \quad (b) q = \frac{4}{3}\pi\rho_0 a^3, \quad (c) q = 2\sigma_0 ab\pi$$

$$15. \theta = \tan^{-1} \left[\frac{Q\sigma_0}{2\epsilon_0 Mg} \right]$$

$$16. (a) E_r = \frac{\lambda L}{2\pi\epsilon_0 r \sqrt{L^2 + r^2}},$$

$$(b) E_x = \frac{\sigma_0}{2\pi\epsilon_0} \left[\sin^{-1} \left(\frac{L^2 - x^2}{L^2 + x^2} \right) + \frac{\pi}{2} \right]; \quad x > 0$$

$$18. (a) E_y = \frac{-\lambda_0 a^2}{\pi \epsilon_0 [z^2 + a^2]^{3/2}}$$

$$(b) E_y = \frac{-\sigma_0}{\pi\epsilon_0} \left\{ \frac{-a}{\sqrt{z^2 + a^2}} + \ln \left[\frac{a + \sqrt{z^2 + a^2}}{|z|} \right] \right\}$$

$$20. (a) E_y = -\frac{\lambda_0 a^2}{\pi\epsilon_0 (a^2 + z^2)^{3/2}}$$

$$21. E_y = \frac{\lambda_0 z^2}{2\pi\epsilon_0 (a^2 + z^2)^{3/2}}$$

$$22. (a) Q_T = 4\pi\epsilon_0 AR^4$$

$$23. (c) E_x = \begin{cases} \frac{\rho_0}{2\epsilon_0 d} (x^2 - d^2) & |x| < d \\ 0 & |x| > d \end{cases}$$

$$25. (c) E_r = \begin{cases} \frac{\rho_0 r^2}{3\epsilon_0 a} & r < a \\ \frac{\rho_0 a^2}{3\epsilon_0 r} & r > a \end{cases}$$

$$26. \mathbf{E} = \frac{\rho_0 d}{2\epsilon_0} \mathbf{i}_x$$

$$27. W = -\frac{\lambda\sigma_0 l^2}{4\epsilon_0}$$

$$28. (a) v_0 \geq \sqrt{\frac{qQ}{2\pi\epsilon_0 Rm}}, \quad (b) r = 4R$$

$$29. (a) \mathbf{E} = -2Ax\mathbf{i}_x, \rho_f = -2A\epsilon_0$$

$$31. (a) \Delta v = \frac{\sigma_0 a}{\epsilon_0}$$

$$32. (a) dq = -\frac{Q}{R} dz'$$

$$33. (c) V \approx \frac{q_0 a}{4\pi\epsilon_0 r^2} \cos \theta, \quad (d) r = r_0 \sin^2 \theta$$

$$34. (d) q_c = -\frac{qV_p}{V_c}$$

$$36. (a) E_y = -\frac{\sigma_0}{2\pi\epsilon_0} \ln\left(1 - \frac{d}{y}\right)$$

$$38. (a) x_0 = \sqrt{\frac{q}{16\pi\epsilon_0 E_0}}, \quad (b) v_0 > \frac{1}{2} \sqrt{\frac{q}{m} \left[\frac{qE_0}{\pi\epsilon_0} \right]^{1/4}}$$

$$(c) W = \frac{q^2}{16\pi\epsilon_0 d}$$

$$43. (e) \lambda = \pm \sqrt{\frac{R_1}{R_2}}, \alpha = \pm \frac{R_1}{R_2}$$

$$44. (g) q_T = -4\pi\epsilon_0 R^2 \frac{\pi^2}{6}$$

Chapter 3

$$2. (a) p_z = \lambda_0 L^2, \quad (e) p_z = QR$$

$$4. (a) \rho_0 = \frac{3Q}{\pi R_0^3}$$

$$7. (a) \mathbf{d} = \frac{4\pi\epsilon_0 R_0^3 \mathbf{E}_0}{Q}$$

$$8. (b) \frac{Q}{L^2} = 2\pi\epsilon_0 E_0$$

$$10. (a) \mathbf{p}_{\text{ind}} = \mathbf{p} \frac{R^3}{D^3}$$

$$12. (a) V(x) = \frac{V_0}{2} \frac{\sinh x/l_d}{\sinh l/l_d}$$

$$15. (b) Q = \frac{m\omega R A \sigma}{q}$$

$$17. (a) D_r = \frac{\lambda}{2\pi r}$$

$$19. (a) \lambda' = -\lambda'' = \frac{\lambda(\epsilon_2 - \epsilon_1)}{\epsilon_1 + \epsilon_2}, \lambda''' = \frac{2\epsilon_2\lambda}{\epsilon_1 + \epsilon_2}$$

$$23. (a) E_r = \begin{cases} -\frac{P_0 r}{\epsilon_0 R} & r < R \\ 0 & r > R \end{cases}$$

$$26. (a) R = \frac{s \ln \frac{\sigma_2}{\sigma_1}}{ID(\sigma_2 - \sigma_1)}$$

$$31. C = \frac{2\pi l(\epsilon_2 a - \epsilon_1 b)}{(b-a) \ln \frac{\epsilon_2 a}{\epsilon_1 b}}$$

$$33. \sigma_f(r=a_1) = \frac{\rho_0 a_0^2}{3a_1} (1 - e^{-u\tau}); \tau = \epsilon/\sigma$$

$$35. \rho_f = \rho_0 e^{-\sigma r^3/(3\epsilon A)}$$

$$38. (a) v(z) = -\frac{V_0 \sinh \sqrt{2RG}(z-l)}{\sinh \sqrt{2RG}l}$$

$$41. (b) \frac{\epsilon\mu}{2}[E^2(l) - E^2(0)] + \epsilon \frac{dv}{dt} = J(t)l$$

$$(c) E(l) = \frac{V_0/l}{1 - \frac{\mu t V_0}{2l^2}}, \quad (f) \tau = \frac{2l^2}{\mu V_0} (1 - e^{-1/2})$$

$$42. (c) E_i^2 = \left(\frac{V_0}{R_0 - R_i}\right)^2 = \frac{I}{2\pi\epsilon\mu l}$$

$$43. (a) W = -\frac{1}{2}\mathbf{p} \cdot \mathbf{E}$$

$$44. W = \frac{p^2}{12\pi\epsilon_0 R^3}$$

$$47. (a) W = \frac{-Q^2}{8\pi\epsilon_0 R}$$

$$48. (a) W_{\text{init}} = \frac{1}{2}CV_0^2, \quad (b) W_{\text{final}} = \frac{1}{4}CV_0^2$$

$$49. (b) W = -pE(\cos \theta - 1)$$

$$50. h = \frac{1}{2}(\epsilon - \epsilon_0) \frac{V_0^2}{\rho_m g^3}$$

$$52. (b) f_y = \frac{1}{2} \frac{\epsilon_0 A}{(s+d)^2} \left[V_0 + \frac{P_0 d}{\epsilon_0} \right]^2$$

$$54. (b) f_z = \frac{\pi V_0^2}{\ln \frac{b}{a}} (\epsilon - \epsilon_0)$$

$$55. f_x = -\frac{1}{2} \frac{\epsilon_0 d}{s} V_0^2$$

$$56. (c) T = \frac{1}{2} v^2 \frac{dC}{d\theta} = \frac{-NV_0^2 R^2 \epsilon_0}{s}$$

$$57. (a) v(t) = \frac{\sigma_f U \omega t}{4\pi \epsilon_0 R}$$

$$58. (a) \rho_f = \rho_0 e^{-\sigma_f / \epsilon U}$$

$$59. (a) nC_i > \frac{1}{R} + \frac{2}{R_L}, \quad (c) \frac{nC_i}{2} > \frac{1}{R}, \quad \omega_0 = \frac{\sqrt{3}}{2} \frac{nC_i}{C}$$

Chapter 4

$$2. (a) V = \begin{cases} \frac{\sigma_0}{2\epsilon a} \cos aye^{-ax} & x > 0 \\ \frac{\sigma_0}{2\epsilon a} \cos aye^{ax} & x < 0 \end{cases}$$

$$4. (a) V = \frac{4V_1}{\pi} \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{\sin \frac{n\pi y}{d} \sinh \frac{n\pi(l-x)}{d}}{n \sinh \frac{n\pi l}{d}}$$

$$7. (a) V_p = \frac{\rho_0}{\epsilon_0 a^2} \sin ax$$

$$12. V(r, \phi) = \begin{cases} \left[\frac{P_2 - P_1}{2\epsilon_0} - E_0 \right] r \cos \phi & 0 \leq r \leq a \\ \left[-E_0 r + \frac{(P_2 - P_1)a^2}{2\epsilon_0 r} \right] \cos \phi & r > a \end{cases}$$

$$13. (a) \mathbf{E} = \left[E_0 \left(1 + \frac{a^2}{r^2} \right) \cos \phi + \frac{\lambda(t)}{2\pi \epsilon r} \right] \mathbf{i}_r - E_0 \left(1 - \frac{a^2}{r^2} \right) \sin \phi \mathbf{i}_\phi$$

$$(b) \cos \phi < -\frac{\lambda(t)}{4\pi \epsilon a E_0}, \quad (c) \lambda_{\max} = 4\pi \epsilon a E_0$$

$$15. (a) V(r, z) = \begin{cases} \frac{V_0 z}{l \ln \frac{b}{a}} \ln \frac{r}{a} & a \leq r \leq b \\ \frac{V_0 z}{l} & b \leq r \leq c \end{cases}$$

$$17. (b) E_0 \geq \sqrt{\frac{8\rho_m g R}{27\epsilon}}$$

$$22. V(2, 2) = V(3, 2) = V(2, 3) = V(3, 3) = -4.$$

$$23. (a) V(2, 2) = -1.0000, V(3, 2) = -.5000, V(2, 3) = -.5000, V(3, 3) = .0000$$

$$(b) V(2, 2) = 1.2500, V(3, 2) = -.2500, V(2, 3) = .2500, V(3, 3) = -1.2500$$

Chapter 5

$$2. (b) B_0^2 > \frac{2mV_0}{es^2}, \quad (e) B_0^2 > \frac{8b^2 m V_0}{e(b^2 - a^2)^2}$$

$$3. B_0 = \frac{-mg}{qv_0}$$

$$4. (d) \frac{e}{m} = \frac{E_y}{RB_0^2}$$

$$8. (c) \mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$10. (a) B_z = \frac{2\mu_0 I (a^2 + b^2)^{1/2}}{\pi ab}, \quad (c) B_z = \frac{n\mu_0 I}{2\pi a} \tan \frac{\pi}{n}$$

$$12. (a) B_z = \frac{\mu_0 K_0 \pi}{4}$$

$$13. (a) B_\phi = \frac{\mu_0 I L}{2\pi r \sqrt{L^2 + r^2}}$$

$$15. (b) B_y = \frac{\mu_0 J_0 d}{2}$$

$$17. (b) B_x = \begin{cases} -\frac{\mu_0 J_0}{2a} (y^2 - a^2) & |y| < a \\ 0 & |y| > a \end{cases}$$

$$18. (d) y = \frac{y_0}{2} \text{ at } x = -\infty$$

$$21. (a) m_z = \frac{1}{2} q \omega a^2$$

23. $\omega_0 = \sqrt{\gamma B_0}$

27. (a) $I' = \frac{(\mu_2 - \mu_1)}{(\mu_1 + \mu_2)} I, I'' = \frac{2\mu_1 I}{\mu_1 + \mu_2}$

34. (a) $\mathbf{H} = \begin{cases} \frac{-M_0}{2} \mathbf{i}_x \\ \frac{M_0 a^2}{2r^2} [\cos \phi \mathbf{i}_r + \sin \phi \mathbf{i}_\phi] \end{cases}$

35. (a) $H_z(x) = -\frac{I}{Dd}(x-d),$ (b) $f_x = \frac{1}{2} \mu_0 \frac{I^2 s}{D}$

36. (a) $f_x = \frac{1}{2}(\mu - \mu_0) H_0^2 D s,$ (b) $f_x = \mu_0 M_0 D s [H_0 + M_0]$

Chapter 6

1. (a) $M = \mu_0 [D - \sqrt{D^2 - a^2}],$ (e) $f_r = \mu_0 I i \left[\frac{D}{\sqrt{D^2 - a^2}} - 1 \right]$

3. (d) $v(t) = v_0 \left[\frac{\alpha}{2\beta} \sin \beta t + \cos \beta t \right] e^{-\alpha t/2}; \beta = \sqrt{\omega_0^2 - \left(\frac{\alpha}{2}\right)^2}$

$$i(t) = \frac{m v_0 \omega_0^2}{B_0 b \beta} \sin \beta t e^{-\alpha t/2}$$

(e) $v_0 > \frac{B_0 b s}{\sqrt{mL}}$

4. (a) $M = \frac{\mu_0 N s}{2\pi} \ln \frac{a}{b}, M = \mu_0 N [R - \sqrt{R^2 - a^2}]$

7. (c) $f_z = \frac{3\mu_0 (I d S)^2}{32\pi d^4}$

8. (a) $H_z = K(t),$ (b) $K(t) = K_0 \left(\frac{x_0}{x_0 - Vt} \right)^{(1-1/R_m)}$

9. (a) $i = \frac{r\sigma d}{2} \frac{dB}{dt} dr,$ (c) $P = \frac{\pi\sigma d a^4}{8} \left(\frac{dB}{dt} \right)^2$

10. $L = \mu_0 N^2 [b - \sqrt{b^2 - a^2}]$

14. (a) $\frac{v_2}{v_1} = \frac{N_2}{2N_1}, \frac{i_2}{i_1} = \frac{2N_1}{N_2}$

16. (a) $V_{oc} = \int_x B_z d \frac{(\mu_+^2 n_+ - \mu_-^2 n_-)}{q(\mu_+ n_+ + \mu_- n_-)^2}$

$$17. (b), (c) \quad EMF = -\frac{\mu_0 V_0 I}{2\pi} \ln \frac{R_2}{R_1},$$

$$(d) \quad EMF = -\frac{(\mu - \mu_0)IV_0}{2\pi} \ln \frac{R_2}{R_1}$$

$$18. (a) \quad \mathbf{H} = 0, \quad \mathbf{B} = \mu_0 M_0 \mathbf{i}_z, \quad (b) \quad v_{oc} = \frac{\omega B_z}{2} (b^2 - a^2)$$

$$20. (b) \quad V > \frac{1}{\mu_0 \sigma N D}$$

$$21. (a) \quad \omega > \frac{(R_r + R_f)}{G}$$

$$(b) \quad C_{crit} = \frac{4L_f}{[R_r + R_f - G\omega]^2}; \quad C > C_{crit}(dc), \quad C < C_{crit}(ac)$$

$$(c) \quad \omega_0 \left[\frac{1}{L_f C} - \left[\frac{R_r + R_f - G\omega}{2L_f} \right]^2 \right]^{1/2}$$

$$22. (b) \quad H_z(x, t) = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{2I_0}{n\pi D} \sin \frac{n\pi x}{d} e^{-\alpha_n t}$$

$$23. (c) \quad H_y(x, t) = H_0 - \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{4H_0}{n\pi} \sin \frac{n\pi x}{d} e^{-\alpha_n t}$$

$$25. (b) \quad H_z(y, t) = -K_0 + \sum_{n=0}^{\infty} \frac{(-1)^n 4K_0}{\pi(2n+1)} \cos \left[\frac{(2n+1)\pi y}{2D} \right] e^{-\alpha_n t}$$

$$(d) \quad \hat{H}_z(y) = K_0 \left[\frac{[e^{(1+j)y/\delta} + e^{-(1+j)y/\delta}]}{[e^{(1+j)D/\delta} + e^{-(1+j)D/\delta}]} \right]$$

$$26. (a) \quad H_z(x) = \frac{K_0}{1 - e^{R_m/l}} [2e^{R_m x/l} - (1 + e^{R_m})]$$

$$27. (a) \quad \hat{H}_z(x) = K_0 e^{-\beta x} e^{R_m x/2l}; \quad \beta = \frac{R_m}{2l} \sqrt{1 + \frac{2jl^2}{R_m^2 \delta^2}}$$

$$28. (a)$$

$$\hat{\mathbf{H}}(x) = \begin{cases} \frac{K_0 \left\{ \left(1 - \frac{\mu}{\mu_0} \frac{k}{\gamma} \right) e^{k(x-s)} (\mathbf{i}_z + j\mathbf{i}_x) + \left(1 + \frac{\mu}{\mu_0} \frac{k}{\gamma} \right) e^{-k(x-s)} (\mathbf{i}_z - j\mathbf{i}_x) \right\}}{\left[1 - \frac{\mu}{\mu_0} \frac{k}{\gamma} \right] e^{-ks} + \left[1 + \frac{\mu}{\mu_0} \frac{k}{\gamma} \right] e^{ks}} & 0 < x < s \\ \frac{2K_0 e^{-\gamma(x-s)} \left[\mathbf{i}_z - \frac{jk}{\gamma} \mathbf{i}_x \right]}{\left[1 - \frac{\mu}{\mu_0} \frac{k}{\gamma} \right] e^{-ks} + \left[1 + \frac{\mu}{\mu_0} \frac{k}{\gamma} \right] e^{ks}} & x > s \end{cases}$$

29. (a) $H_y = H_0 \frac{\cosh k(x-d/2)}{\cosh kd/2}$
32. (b) $\hat{H}_\phi(r) = \frac{I_0}{2\pi a} \frac{J_1[(r/\delta)(1-j)]}{J_1[(a/\delta)(1-j)]}$
33. (a) $T = -L_1 I_0^2 \cos^2 \omega_0 t \sin 2\theta$
34. (c) $T = M_0 I_1 I_2 \cos \theta$,
- (f) $\theta(t) = \theta_0 \left[\cos \beta t + \frac{\alpha}{2\beta} \sin \beta t \right] e^{-\alpha t/2}$
35. (a) $L(x) = \frac{\mu_0 x}{2\pi} \ln \frac{b}{a}$, (b) $f_x = \frac{\mu_0 i^2}{4\pi} \ln \frac{b}{a}$
37. $h = \frac{I^2(\mu - \mu_0) \ln \frac{b}{a}}{4\pi^2 \rho_m g (b^2 - a^2)}$

Chapter 7

4. (b) $W = 4[P_s E_c + \mu_0 M_s H_c]$
9. (b) $\hat{E}_x(z) = \begin{cases} \frac{j\eta_0 J_0 \sin kd e^{-j\eta_0(z-d)}}{\omega\epsilon[\eta_0 \sin kd - j\eta \cos kd]} & z > d \\ \frac{J_0 \eta \cos kz}{\omega\epsilon[\eta_0 \sin kd - j\eta \cos kd]} & z < -d \\ -\frac{J_0 \eta \cos kz}{\omega\epsilon[\eta_0 \sin kd - j\eta \cos kd]} & |z| < d \end{cases}$
10. (b) $E_x = E_0 e^{j(\omega t \mp \beta z)} e^{\mp(\alpha z/2)}$ $\begin{matrix} z > 0 \\ z < 0 \end{matrix}$; $\beta = \sqrt{\omega^2 \epsilon \mu - \frac{\alpha^2}{4}}$
11. (a) $t'_1 - t'_2 = \frac{\gamma v}{c_0^2} (z_2 - z_1)$,
- (b) $t'_1 - t'_2 = \gamma(t_1 - t_2)$, (c) $z'_2 - z'_1 = \gamma L$
12. (a) $u'_z = \frac{\dot{u}_z - v}{1 - vu_z/c_0^2}$, $u'_{x,y} = \frac{u_{x,y} \sqrt{1 - (v/c_0)^2}}{1 - vu_z/c_0^2}$
15. (b) $\epsilon(\omega) = \epsilon_0 \left[1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2} \right]$
16. (c) $k^2 = \frac{\omega^2}{c^2} \left[1 - \frac{\omega_p^2}{\omega(\omega \mp \omega_0)} \right]$
20. (a) $\mathbf{E} = E_0 \left[\cos \omega \left(t - \frac{z}{c} \right) - \cos \omega \left[\left(1 - \frac{2v}{c} \right) \left(t - \frac{z}{c} \right) \right] \right] \mathbf{i}$,
22. $\alpha^2 - k^2 = -\omega^2 \epsilon \mu$, $\alpha \cdot \mathbf{k} = \frac{1}{\delta^2}$

$$26. (a) L_1 + L_2 = s_i \sin \theta_i + s_r \sin \theta_r = h_1 \tan \theta_i + h_2 \tan \theta_r$$

$$31. \theta_i \approx 41.7^\circ$$

$$33. (a) \left(\frac{x}{R}\right)^2 > 1 - \frac{(n^2 - 2)^2}{4}; \sqrt{2} \leq n \leq 2$$

$$(b) R' = \frac{\alpha R}{[\sqrt{n^2(1-\alpha^2)}\sqrt{n^2-\alpha^2} + \alpha^2]}$$

Chapter 8

$$2. (c) \hat{v}(z) = -\frac{V_0 \sin \beta(z-l) e^{\alpha z/2}}{\sin \beta l} \quad (\text{Short circuited end})$$

$$4. (c) \omega^2 = \omega_p^2 + k^2 c^2, \quad (d) v(z, t) = -\frac{V_0 \sin kz \cos \omega t}{\sin kl}$$

$$5. (b) k = \frac{1}{\omega \sqrt{LC}}$$

$$14. (a) V_+ = -V_- = \frac{V_0 Z_0}{2R_s}$$

$$16. (b) \tan kl = -XY_0$$

$$21. (c) VSWR = \frac{1 + \sqrt{2}}{\sqrt{2} - 1} \approx 5.83$$

$$22. (b) VSWR = 2$$

$$23. Z_L = 170.08 - 133.29j$$

$$24. (a) l_1 = .137\lambda + \frac{n\lambda}{2}, l_2 = .089\lambda + \frac{m\lambda}{2}$$

$$l_1 = .279\lambda + \frac{n\lambda}{2}, l_2 = .411\lambda + \frac{m\lambda}{2}$$

$$25. (a) l_1 = .166\lambda + \frac{n\lambda}{2}, l_2 = .411\lambda + \frac{m\lambda}{2}$$

$$l_1 = .077\lambda + \frac{n\lambda}{2}, l_2 = .043\lambda + \frac{m\lambda}{2}$$

$$27. (e) \alpha = \frac{2(\pi/a)^2 [b + (a/2)(\omega^2 a^2 / \pi^2 c^2)]}{\omega \mu a b k_x \sigma_w \delta}$$

$$28. (b) = \frac{2\omega \epsilon (b k_x^2 + a k_y^2)}{\sigma_w \delta k_x a b (k_x^2 + k_y^2)}$$

29. (a) TE mode:

$$\text{electric field: } \cos k_x x \cos k_y y = \text{const}$$

$$\text{magnetic field: } \frac{\sin(k_x x)^{(k_y/k_x)^2}}{\sin k_y y} = \text{const}$$

$$31. (b) \frac{\omega^2}{c^2} = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{l}\right)^2,$$

$$32. (a) \omega^2 = \frac{k_x^2}{\epsilon\mu - \epsilon_0\mu_0}$$

Chapter 9

$$1. (c) \hat{V} = \frac{\hat{Q}}{4\pi\epsilon r} e^{-jr\sqrt{\omega^2/c^2 - j\omega\mu\sigma}}$$

$$4. (a) dl_{\text{eff}} = \frac{2 \sin(\alpha dl/2)}{\alpha}, \quad \hat{\lambda}(z) = \frac{I_0 \alpha}{j\omega} \sin \alpha z$$

$$6. (a) \hat{p}_z = 4\pi\epsilon_0 R^3 \hat{E}_0$$

$$(c) \langle P \rangle = \frac{\omega^4 |\hat{p}_z|^2 \eta}{12\pi c^2}$$

$$7. (a) m_{\text{ind}} = 2\pi H_0 R^3$$

$$8. (b) \sin^2 \theta \left[\frac{\cos(\omega t - kr)}{kr} - \sin(\omega t - kr) \right] = \text{const}$$

$$9. (a) \hat{E}_\theta \approx \frac{2\hat{E}_0}{jkr} \sin \theta e^{-j(kr - \chi/2)} \left[\cos \left(ka \cos \theta - \frac{\chi}{2} \right) \right]$$

$$11. \hat{E}_\theta = \frac{2jK_0 dl \eta e^{-jkr}}{4\pi r \cos \phi} \sin \left(\frac{kL}{2} \sin \theta \cos \phi \right)$$

$$12. (a) \hat{E}_\theta = \frac{\eta I_0 \sin \theta e^{-jkr}}{j\pi kr L \cos^2 \theta} \cos \left[\left(\frac{kL}{2} \cos \theta \right) - 1 \right]$$