

**Experiment 11: Driven *RLC* Circuit****OBJECTIVES**

1. To measure the resonance frequency and the quality factor of a driven *RLC* circuit by creating a resonance (frequency response) curve.
2. To see the phase relationships between driving voltage and driven current in such a circuit at, below, and above the resonance frequency.
3. To use *DataStudio* display capabilities to carry out these objectives.

**INTRODUCTION**

The presence of inductance in an electric circuit gives the current an “inertia” (resistance to change), since inductors try to prevent changes in the flow of current. The presence of capacitance in a circuit means that charge can flow into one side of the capacitor to be stored there, and later this charge can restore the electric current as the capacitor discharges. These two properties of inertia and energy storage are analogous to the inertia and energy storage of a mass-spring combination, which you studied in mechanics. In a mechanical system viscous friction causes damping, and in electric circuits resistance causes the damping.

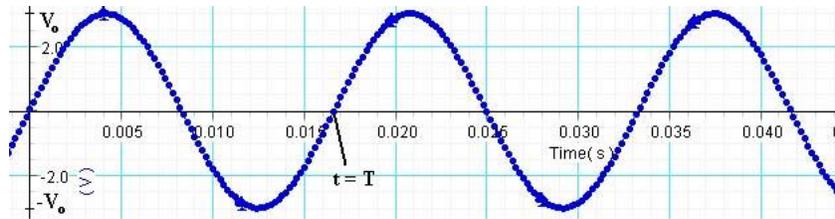
If a mechanical system that has a natural frequency of oscillation is “driven” by a periodic external force whose frequency matches the natural frequency of oscillations, then the system is said to be “in resonance” with the driving force and the amplitude of oscillations can grow very large. An electric circuit driven by a periodic external voltage exhibits the same behavior. In this experiment, you will study the properties of circuits consisting of an inductor, capacitor and resistor in series. You will observe the behavior near resonance and measure the resonant frequency.

**Driven *RLC* Circuit**

Suppose we have an AC (Alternating-Current) voltage source given by

$$V(t) = V_0 \sin\left(\frac{2\pi t}{T}\right) = V_0 \sin(2\pi f t) = V_0 \sin(\omega t) \quad (11.1)$$

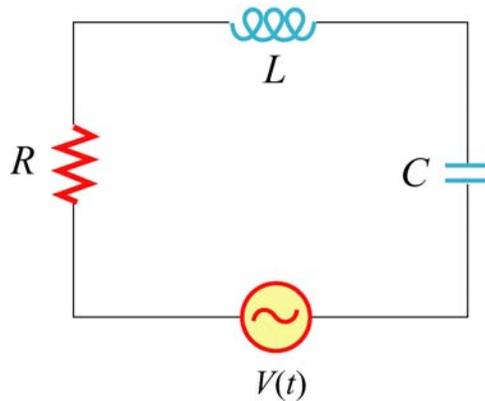
where  $V_0$  is the *amplitude* (maximum value). The voltage varies between  $V_0$  and  $-V_0$  since a sine function varies between +1 and -1. A typical graph of voltage as a function of time is shown in Figure 1.



**Figure 1** Sinusoidal voltage source

The sine function is periodic in time. This means that the value of the voltage at time  $t$  will be exactly the same at a later time  $t' = t + T$  where  $T$  is the *period*. The *frequency*,  $f$ , is defined to be the reciprocal of the period,  $f = 1/T$ . The units of frequency are inverse seconds,  $s^{-1}$ , which are called hertz [Hz]. The angular frequency  $\omega$  is defined to be  $\omega = 2\pi f$ .

Suppose the voltage source is connected in a series circuit consisting of a coil with self-inductance  $L$ , a resistor of resistance  $R$  and a capacitor with capacitance  $C$ , as shown in Figure 2.



**Figure 2** Driven  $RLC$  Circuit

The expression for the current in the series circuit as a function of time is derived in the 8.02 *Course Notes*, **Section 12.3**. The result is reproduced here as

$$\begin{aligned}
 i(t) &= I_0(\omega) \sin(\omega t - \phi), \\
 I_0(\omega) &= \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}, \\
 \tan \phi(\omega) &= \frac{\omega L - \frac{1}{\omega C}}{R}
 \end{aligned}
 \tag{11.2}$$

(these experiment instructions will use lowercase  $i(t)$  and  $q(t)$  for time-varying currents and charges, but  $V(t)$  for time-varying voltages). Note that we explicitly indicate the dependence of these quantities on the driving frequency  $\omega$ . This quantity is something we get to choose in this

experiment, and once we select the frequency we can calculate how the circuit responds using Equations (11.2).

The amplitude  $I_0(\omega)$  is at a maximum when the term in parentheses in the denominator vanishes:

$$\omega L - \frac{1}{\omega C} = 0. \quad (11.3)$$

We can solve Equation (11.3) for the driving frequency and see that resonance occurs at

$$\omega_{\text{maximum current}} = \omega_0 = \frac{1}{\sqrt{LC}} \quad (11.4)$$

(see the 8.02 *Course Notes*, **Section 11.5** for a further discussion of resonance).

The amplitude at resonance is then

$$I_{0,\text{max}} = V_0/R \quad (11.5)$$

and  $\phi = 0$  at resonance since  $\tan^{-1} 0 = 0$ . The corresponding current response is given by

$$i(t) = \frac{V_0}{R} \sin \omega_0 t. \quad (11.6)$$

The power delivered to the circuit by the voltage source is

$$P(t) = V(t) i(t) = \frac{V_0^2}{R} \sin^2 \omega_0 t \quad (11.7)$$

and its time-averaged is

$$\langle P(t) \rangle = \frac{1}{T} \int_0^T P(t) dt = \frac{1}{T} \int_0^T \frac{V_0^2}{R} \sin^2 \omega_0 t dt = \frac{V_0^2}{2R}, \quad (11.8)$$

which is the same as the Joule heating in the circuit;

$$\langle P(t) \rangle = \frac{1}{T} \int_0^T i^2 R dt = \frac{1}{T} \int_0^T \frac{V_0^2}{R} \sin^2 \omega_0 t dt = \frac{V_0^2}{2R}. \quad (11.9)$$

Thus, all the power introduced into the circuit is dissipated in the resistor.

When the driving angular frequency is such that  $\omega < \omega_0 = \frac{1}{\sqrt{LC}}$ , then  $\omega L - \frac{1}{\omega C} < 0$ , and

$$\phi(\omega) = \tan^{-1} \left( \frac{\omega L - \frac{1}{\omega C}}{R} \right) < 0 \quad (11.10)$$

A negative phase shift means that circuit behaves in a *capacitive* fashion: the driven current leads the driving voltage. When the oscillations take a very long time to complete a cycle, the charge builds up on the plate. Hence, the charged capacitor will determine the current the flow of current.

On the other hand, when  $\omega > \omega_0$ , the phase shift is positive,  $\phi(\omega) > 0$ . This implies that circuit behaves in an *inductive* fashion: the driven current lags the driving voltage. This shouldn't be surprising because the oscillations are getting very rapid, which means the self-inductance dominates.

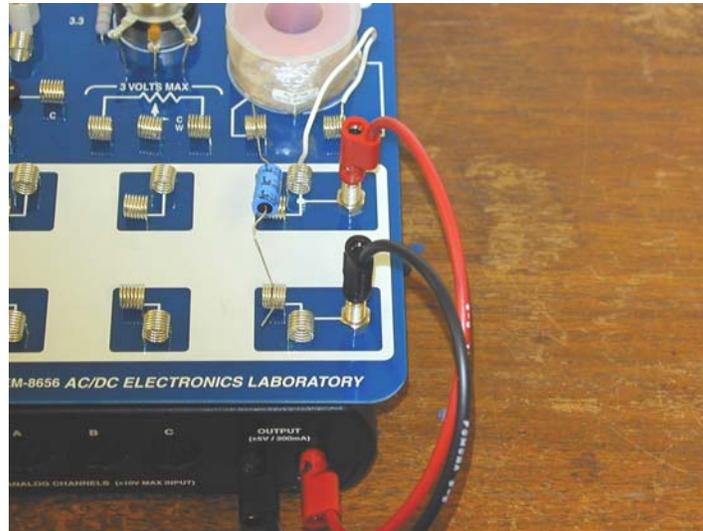
## EXPERIMENTAL SETUP

### Components needed

- 100- $\mu$ F capacitor
- 750 Interface
- AC/DC Electronics Lab Circuit Board
- Two Data Studio files: **exp11a.ds**, **exp11b.ds**.

**Computer Setup:** If not already done, connect the 750 Interface to the computer using the SCSI cable. Connect the power supply to the 750 Interface and turn on the interface power. Always turn on the interface before powering up the computer. Turn on your computer.

**AC/DC Electronics Lab Circuit Board:** On your AC/DC circuit board, place the 100- $\mu$ F capacitor in series with the inductor coil on the electronics board (make sure the iron core is removed from the inductor). Connect the **Signal Generator** of the 750 Interface into the electronics board using the banana plugs (see Figure 3), and connect the **Signal Generator** output in series with the capacitor and the inductor coil. Note that in Figure 3, one capacitor lead is connected to the same port as the black lead from the output, but the other is connected to the inductor coil lead, *not* the red lead from the output.



**Figure 3** Setup of the AC/DC Electronics Lab Circuit Board

**Data studio Files:** Download the file **exp11a.ds** from the web page. This file has a Signal Generator Display and two scope displays (see figure below). In the Signal Generator dialog box make sure that a Sine Wave Function has been selected. If necessary, adjust the Amplitude to 1.7 V, the Frequency to 20 Hz and the Sampling Rate to 10000 Hz .

**Question 1 (answer on the tear-sheet at the end):** As you determined in a previous lab, the resistance of the coil is between  $5\ \Omega$  and  $10\ \Omega$  and its inductance is approximately  $8.5\ \text{mH}$  . Calculate the resonant frequency you should expect for your experiment in hertz (cycles per second)

using the formula  $f_0 = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$ , with  $L = 8.5\ \text{mH}$  and  $C = 100\ \mu\text{F}$  .

## MEASUREMENTS

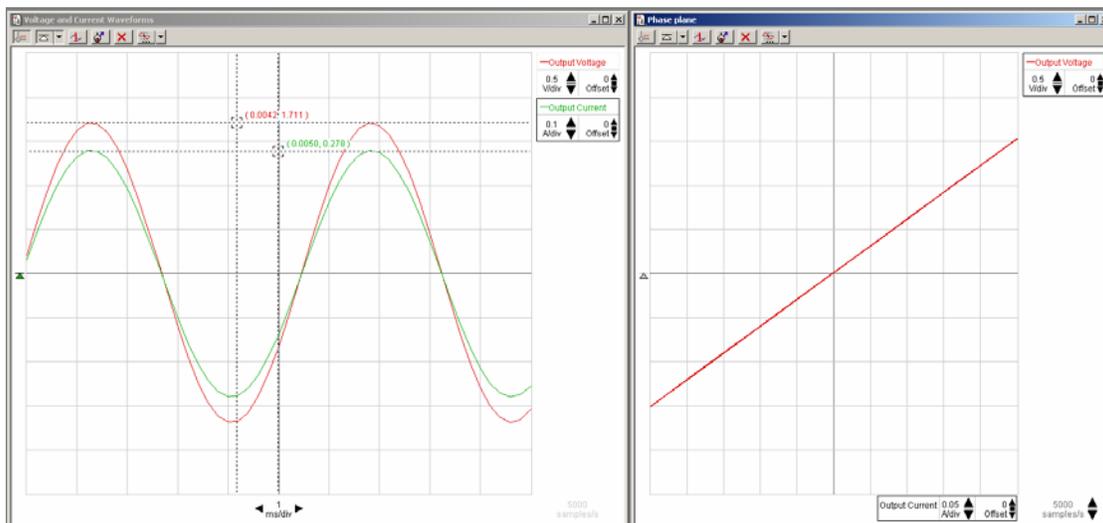
### Part 1: Resonant Frequency

Click **Start** to begin collecting data. On the **Voltage and Current Waveform** scope display, you should see two waveforms for the **Output Voltage** and the **Output Current**, as shown on the left in Figure 4 (if necessary, click the **Trigger** control, second icon from the left, on the **Scope** display to stabilize the picture). You can adjust the display using the controls at the upper right or at the bottom of the display window. You might find that it helps to get a more stable display if you lower the voltage amplitude slightly.

The scope display on the right shows **Output Voltage** as a function of **Output Current**. Such a plot is known as a “phase-plane plot,” for reasons that we will see explicitly.

Increase the frequency of the sine wave in 10-Hz steps from 20 Hz to 200 Hz (change the frequency in the **Signal Generator** window). Watch both of the scope displays and see if you can determine how the shape of the ellipse in the phase-plane plot is related to the current and voltage waveforms.

Identify a range of frequencies where the Output Current seems to have a maximum. Then use smaller frequency steps in order to locate the resonant frequency more precisely.



**Figure 4** Waveform scope display (left) and phase plane scope display (right)

**Question 2 (answer on the tear-sheet at the end):** Record the resonant frequency at which the Output Current reaches a maximum. Does your value agree with the theoretical value you calculated above in Question 1?

**Question 3 (answer on the tear-sheet at the end):** How does the phase relation between Output Current and Output Voltage change as you increase the frequency from below to above the resonant frequency? How could you use this information to help pinpoint the resonant frequency?

**Question 4 (answer on the tear-sheet at the end):** Observe what happens in the phase-plane plot as you pass through the resonant frequency of the circuit. When you are exactly at resonance, the scope should display a straight line (a degenerate ellipse). Can you explain this behavior?

Use the SmartTool to measure the amplitude of the Output Current and Output Voltage at the resonant frequency.

**NOTE on using the SmartTool in the Scope Display:** When you have more than one signal being displayed on the scope, you can have a SmartTool for *each* trace. You need separate SmartTools because each signal will have a different scale. Look at the right side of the display. Click the name of the signal you want to make active and a box will appear around it. Then click on the SmartTool button and a SmartTool cursor will appear in the color of that trace. You can do the same for the other trace.

**Question 5 (answer on the tear-sheet at the end):** Use your measurements in Equation (11.5) to determine the resistance (in ohms) of the inductor coil.

There is a display in this *DataStudio* activity that gives the ratio of the maximum voltage recorded to the maximum current recorded (this is the *impedance*, the term in the square root in Equation (11.2))

As the frequency is increased to the resonant frequency, the impedance decreases. The minimum impedance is expected to be the resistance. Check to see if this is true.

Since the Output Voltage and Output Current are *in phase* with each other at resonance, the output power  $P$  varies as the square of a sinusoidal function. The maximum value of the output power is  $P_{\max} = I_{\max} V_{\max}$  and the *average* power supplied is one half of the maximum (because the average value of  $\sin^2$  is  $\frac{1}{2}$ ).

**Question 6 (answer on the tear-sheet at the end):** Compute the *average* power (in watts) consumed by your circuit at resonance.

There is a display in this *DataStudio* activity that gives the product of the maximum voltage recorded and the maximum current recorded, divided by 2 (this window title is not entirely accurate). The maximum value attained should be the average value at resonance. Check to see that this is true.

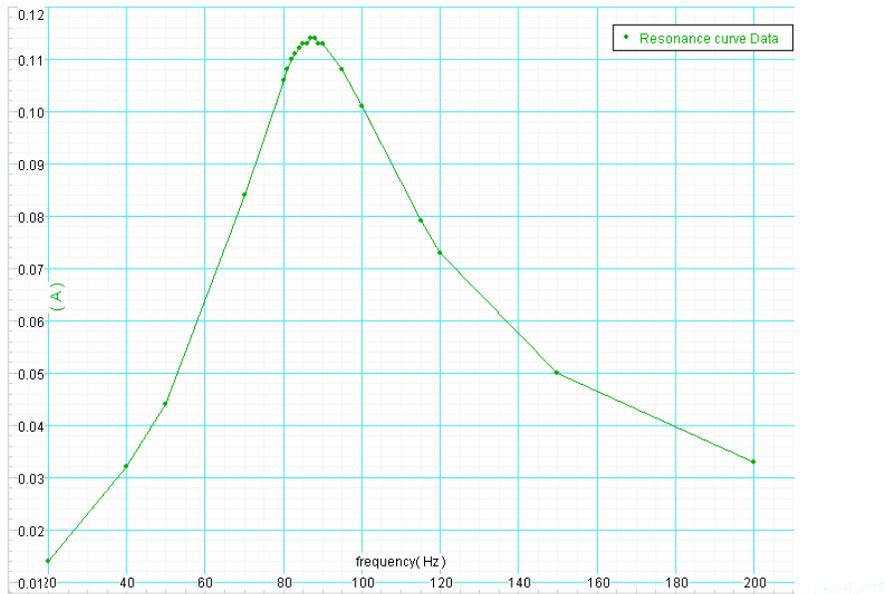
**Interesting extra observation:** When the circuit is at or near resonance, insert the iron core into the coil. Can you explain the behavior of the phase-plane plot?

## Part 2: Resonance Curve and the Quality Factor

Now examine the behavior of the circuit near resonance in more detail. For this purpose, it is more convenient to use a **Graph** display than the scope, so download and open the file **exp11b.ds**, which should simplify data collection. There are several windows, not all of which you need at the same time. Starting at 20 Hz on the **Signal Generator**, click **Start** to obtain the maximum value of the output current for this frequency. As the data is taken, this maximum value is measured and automatically placed in the box identifying the data run. Record the maximum value of the current for this 20-Hz frequency in the **Resonance Data Table** on the right, in the corresponding row of the column to the right of the frequency column. Then increase the frequency of the Signal Generator to the next value of the frequency in the **Resonance Data Table** and again record the maximum current in the data table. As you enter data in the table, a curve will be drawn in the **Resonance Curve** window. The curve will look somewhat like Figure 5; the figure shown is for a circuit with different parameters, and with fewer measurements away from resonance.

Continue doing the measurements of current amplitude from 20 Hz to 300 Hz and adding to the table. If you want to better define the shape of the curve around the resonance, take extra data points near the resonant frequency so you can see clearly what happens to the current amplitude near resonance (this was done in preparing Figure 5). To add frequencies, you will need to insert extra rows in the **Resonance Data Table**; this is done with the second icon from the left. Extra rows should be entered in order of increasing frequency.

**Question 7 (answer on the tear-sheet at the end):** From your Resonance Curve, what is the resonant frequency? How does this compare to your answers to Questions 1 and 2?



**Figure 5** Resonance curve

The **quality factor** of a circuit is a measure of how sharp the resonance curve is. For our purposes, the quality  $Q$  (not to be confused with charge) is defined in terms of the power output,

$$Q \equiv \frac{\omega}{\Delta\omega} = \frac{\omega_0}{\omega_+ - \omega_-}, \quad (11.11)$$

where  $\omega_0$  is the resonant frequency and  $\omega_+$  and  $\omega_-$  are the frequencies for which the average power drops to  $\frac{1}{2}$  of its value at  $\omega_0$  (see the 8.02 Course Notes, Section 12.4).

For ease in determining the width of the power versus frequency curve, a plot of the square of the current as a function of frequency is included in the *DataStudio* activity.

**Question 8 (answer on the tear-sheet at the end):** Compute  $Q$  for your circuit by measuring the full width at half max on the  $I^2$  versus frequency curve and computing

$$Q = \frac{\omega_0}{\Delta\omega_{\text{full width at half max}}} = \frac{f_0}{\Delta f_{\text{full width at half max}}}. \quad (11.12)$$

Compare to the predicted value of

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0 L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}. \quad (11.13)$$

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
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8.02

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**Experimental Summary 11: Driven *RLC* Circuit**

**Group** \_\_\_\_\_

**Names** \_\_\_\_\_

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**Question 1:** Calculate the resonant frequency you should expect for your experiment in hertz (cycles per second) using the formula  $f_0 = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$ , with  $L = 8.5$  mH and  $C = 100\mu\text{F}$ .

**Answer:** \_\_\_\_\_ Hz

**MEASUREMENTS**

**Question 2:** Record the resonant frequency at which the **Output Current** reaches a maximum. Does your value agree with the theoretical value you calculated above?

**Answer:** \_\_\_\_\_ Hz

**Question 3:** How does the phase relation between **Output Current** and **Output Voltage** change as you increase the frequency from below to above the resonant frequency? How could you use this information to help pinpoint the resonant frequency?

**Answer:**

**Question 4:** Observe what happens at the resonance frequency of the circuit. When you are exactly at resonance, the scope should display a straight line. Can you explain this behavior?

**Answer:**

**Question 5:** Use your measurements in Equation (11.5) to determine the resistance (in ohms) of the inductor coil.

**Answer:** \_\_\_\_\_  $\Omega$

**Question 6:** Compute the *average* power (in watts) consumed by your circuit at resonance.

**Answer:** \_\_\_\_\_ W

### Part 2: Resonance Curve and the Quality Factor

**Question 7:** From your Resonance Curve, what is the resonant frequency? How does this compare to your answers to Questions 1 and 2?

**Answer:** \_\_\_\_\_ Hz

**Question 8:** Compute  $Q$  for your circuit by measuring the full width at half max on the  $I^2$  versus frequency curve and computing  $Q = \frac{\omega_0}{\Delta\omega_{\text{full width at half max}}} = \frac{f_0}{\Delta f_{\text{full width at half max}}}$ .

**Answer:** \_\_\_\_\_ (trick question: What are the units of  $Q$ ?)

Compare to the predicted value using the formula

$$Q = \frac{\omega_0 L}{R}.$$

**Answer:** \_\_\_\_\_