

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Problem Solving 7: Faraday's Law

OBJECTIVES

1. To explore a particular situation that can lead to a changing magnetic flux through the open surface bounded by an electric circuit.
2. To calculate the rate of change of magnetic flux through the open surface bounded by that circuit in this situation.
3. To determine the sense of the induced current in the circuit from Lenz's Law.
4. To look at the forces on the current carrying wires in our circuit and determine the effects of these forces on the dynamics of the circuit.

REFERENCE: Sections 10.1 – 10.2, 8.02 Course Notes.

Problem-Solving Strategy for Faraday's Law

In Chapter 10 of the 8.02 Course Notes, we have seen that a changing magnetic flux induces an emf:

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

according to Faraday's law of induction. For a conductor that forms a closed loop, the emf sets up an induced current $I = |\mathcal{E}|/R$, where R is the resistance of the loop. To compute the induced current and its direction, we follow the procedure below:

(1) For the closed loop of area A , define an area vector \vec{A} and choose a positive direction. For convenience of applying the right-hand rule, let \vec{A} point in the direction of your thumb. Compute the magnetic flux through the loop using

$$\Phi_B = \begin{cases} \vec{B} \cdot \vec{A} & (\vec{B} \text{ is uniform}) \\ \iint \vec{B} \cdot d\vec{A} & (\vec{B} \text{ is non-uniform}) \end{cases}$$

Determine the sign of Φ_B . We choose $\Phi_B > 0$ to be the outward flux and $\Phi_B < 0$ to be the inward flux.

(2) Evaluate the rate of change of magnetic flux $d\Phi_B/dt$. Keep in mind that the change could be caused by

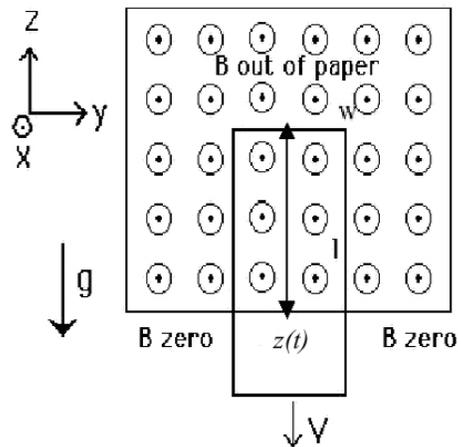
- (i) changing the magnetic field $dB/dt \neq 0$,
- (ii) changing the loop area if the conductor is moving ($dA/dt \neq 0$), or
- (iii) changing the orientation of the loop with respect to the magnetic field ($d\theta/dt \neq 0$).

Determine the sign of $d\Phi_B/dt$.

(3) The sign of the induced emf is opposite the sign of $d\Phi_B/dt$. The direction of the induced current can be found by using the Lenz's law discussed in Section 10.2.2.

Example: A Falling Loop

A rectangular loop of wire with mass m , width w , vertical length l , and resistance R falls out of a magnetic field under the influence of gravity. The magnetic field is uniform and out of the paper ($\mathbf{B} = B\hat{x}$) within the area shown (see sketch) and zero outside of that area. At the time shown in the sketch, the loop is exiting the magnetic field at speed $\vec{V}(t) = V(t)\hat{z}$, where $V(t) < 0$ (meaning the loop is moving downward, not upward). Suppose at time t the distance from the top of the loop to the point where the magnetic field goes to zero is $z(t)$ (see sketch).



Question 1 (*write your answer on the tear-off sheet at the end!!!*): What is the relationship between $V(t)$ and $z(t)$? Be careful of you signs here, remember that $z(t)$ is positive and decreasing with time, so $dz(t)/dt < 0$.

Problem Solving Strategy Step (1): Define \vec{A} and Compute Φ_B

Question 2: If we define the area vector \vec{A} to be out of the page, what is the magnetic flux Φ_B through our circuit at time t ? Be careful, because the magnetic field is not uniform across the entire area of the loop! (it is zero over some parts and constant and non-zero over other parts).

Problem Solving Strategy Step (2): Compute $d\Phi_B / dt$

Question 3: What is $d\Phi_B / dt$? Is this positive or negative at time t ? Be careful here, your answer should involve $V(t)$, and remember that $V(t) < 0$.

Problem Solving Strategy Step (3): Determine the sign of the induced emf (the same as the direction of the induced current)

Question 4: If $d\Phi_B / dt < 0$ then your induced emf (and current) will be right-handed with respect to \vec{A} , and vice versa. What is the direction of your induced current given your answer to Question 3, clockwise or counterclockwise?

Question 5: Lenz's Law says that the induced current should be such as to create a self-magnetic field (due to the induced current alone) which tries to keep things from changing. What is the direction of the self-magnetic field due to your induced current inside the circuit loop, into the page or out of the page? Is it in a direction so as to keep the flux through the loop from changing?

Question 6: What is the magnitude of the current flowing in the circuit at the time shown (use $I = |\varepsilon| / R$)?

Question 7: Besides gravity, what other force acts on the loop in the $\pm \hat{z}$ direction? Give the magnitude and direction of this force in terms of the quantities given (hint: $d\vec{F} = I d\vec{s} \times \vec{B}$).

Question 8: Assume that the loop has reached "terminal velocity"--that is, that it is no longer accelerating. What is the magnitude of that terminal velocity in terms of given quantities?

Question 9: What is the general equation that states that at terminal velocity, the rate at which gravity is doing work on the loop is equal to the rate at which energy is being dissipated in the loop through Joule heating (the rate at which a force does work is $\vec{F} \cdot \vec{V}$)?

Question 10: Show that your general equation from Question 9 is in fact true in this case by inserting into that general equation your current in Question 7 and your terminal velocity in Question 8.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Tear off this page and turn it in at the end of class !!!!

Note:
Writing in the name of a student who is not present is a Committee on Discipline offense.

Problem Solving 7: Faraday's Law

Group _____ (e.g. 6A Please Fill Out)

Names _____

Question 1: What is the relationship between $V(t)$ and $z(t)$? _____

Question 2: What is the magnetic flux Φ_B through our circuit at time t ? _____

Question 3: What is $d\Phi_B / dt$? _____

Is this positive or negative at time t ? _____

Question 4: What is the direction of your induced current given your answer to Question 3, clockwise or counterclockwise? _____

Question 5: What is the direction of the self-magnetic field due to your induced current inside the circuit loop, into or out of the page? _____

Is it in a direction so as to keep the flux through the loop from changing? _____

Question 6: What is the current flowing in the circuit at the time t ? _____

Question 7: Besides gravity, what other force acts on the loop in the $\pm \hat{z}$ direction? Give the magnitude and direction of this force in terms of the quantities given.

Question 8: What is the terminal velocity in terms of given quantities?

Question 9: What is the general equation that says that at terminal velocity, the rate at which gravity is doing work on the loop is equal to the rate at which energy is being dissipated in the loop through Joule heating?

Question 10: Show that your general equation from Question 9 is in fact true in this case by inserting into that general equation your answers in Question 7 and Question 8.
