

vacuum is considered more reliable than any value previously reported.

In measuring coefficient of linear expansion the total length observable with the comparator was only about 4 mm. Temperature measurements could be made only at room temperature and in the pyrometer range. Transition points, if they exist, lie between these limits. Repeated readings were not satisfactorily consistent. Doubtless the peculiar properties of uranium described at the beginning of this section partially account for the fact. The average value obtained for the coeffi-

cient of linear expansion over the range of 25°C to 1000°C is 4×10^{-5} , which is exceptionally large. No other value has appeared in the literature.

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Electronic Energy Bands in Metallic Tungsten¹

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Calculations of the electronic energy bands in metallic tungsten are carried out by the Wigner-Seitz-Slater method. All numerical integrations were carried out on the M. I. T. differential analyzer. It is found that the d band is broken up into five sub-bands. Some of these d bands are found to be about fifteen electron volts in width. One is about two electron volts in width. The occupied energy range extends about five electron volts. The s band starts at a higher energy than the d bands and is occupied by much less than one electron per atom at the equilibrium interatomic distance. From the results of a previous paper, curves of E vs. k are plotted for the principal directions of propagation. From these, curves of the number of energy levels per unit energy range were determined by numerical and graphical methods which are described in an appendix. It is assumed that the $n(E)$

curve for tantalum is not greatly different from that for tungsten except that there is one less electron per atom. From the $n(E)$ curves the electronic contribution to the specific heat is calculated for the two metals and the results found to be in good agreement with the excess specific heat at high temperatures for both metals. The computed value does not agree with low temperature data on tantalum. There are no low temperature data for tungsten. Qualitative discussions of the differences in electrical resistance, temperature coefficient of resistance, and thermoelectric power of the two metals are given. The contribution of exchange effects to the paramagnetic susceptibility is treated by a rough model and it is shown that the assumption of the same value of the exchange integral for both metals gives satisfactory agreement with observed data.

CALCULATIONS of the electronic energy bands in solids have been carried out by either the Wigner-Seitz cellular method or by Slater's modification of that method for monovalent metals, for calcium, and for a number of insulators.^{2, 3} No calculations by either of these

methods have been reported for any transition metals, although some general characteristics have been discussed by Mott^{4, 5, 6} in a series of

¹ A preliminary report of this work was presented at the Washington Meeting of the American Physical Society in April, 1938. Phys. Rev. **53**, 673 (1938).

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