Statistics in Materials Testing

- Basic statistical measures

$$\text{arithmetic mean} \quad \overline{\sigma_f} = \frac{1}{N}\sum_{i=1}^{N}\sigma_{f,i}$$

$$\text{standard deviation} \quad s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\overline{\sigma_f} - \sigma_{x,i}\right)^2}$$

Room-temperature tensile strength of a graphite/epoxy composite (P. Shyprykevich, *ASTM STP 1003,* pp. 111–135, 1989.) (in kpsi): 72.5, 73.8, 68.1, 77.9, 65.5, 73.23, 71.17, 79.92, 65.67, 74.28, 67.95, 82.84, 79.83, 80.52, 70.65, 72.85, 77.81, 72.29, 75.78, 67.03, 72.85, 77.81, 75.33, 71.75, 72.28, 79.08, 71.04, 67.84, 69.2, 71.53.

$$\overline{\sigma_f} = 73.28, \qquad s = 4.63 \text{ (kpsi)}$$

The coefficient of variation is C.V.= $(4.63/73.28) \times 100\% = 6.32\%$.

- The normal distribution

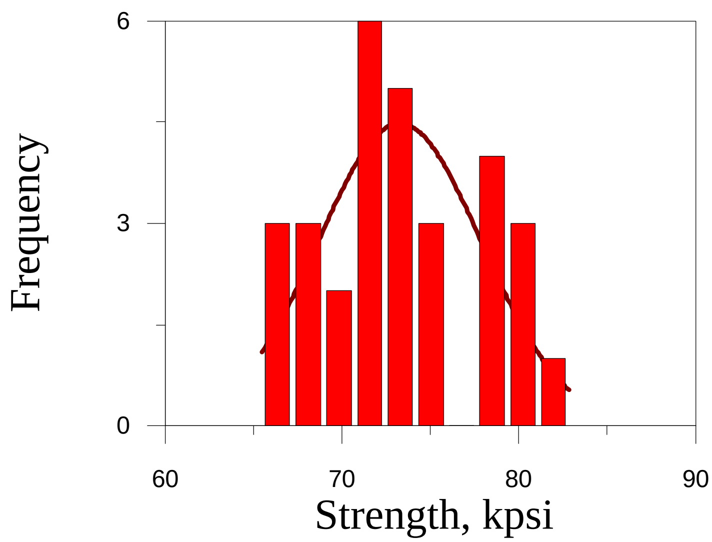

Figure 1: Histogram and normal distribution functions.

$$f(X) = \frac{1}{\sqrt{2\pi}} \exp \frac{-X^2}{2}, \quad X = \frac{\sigma_f - \overline{\sigma_f}}{s}$$

Cumulative probability

$\pm x/s$	%
1	68.3
1.96	95.0
2	95.8
3	99.7

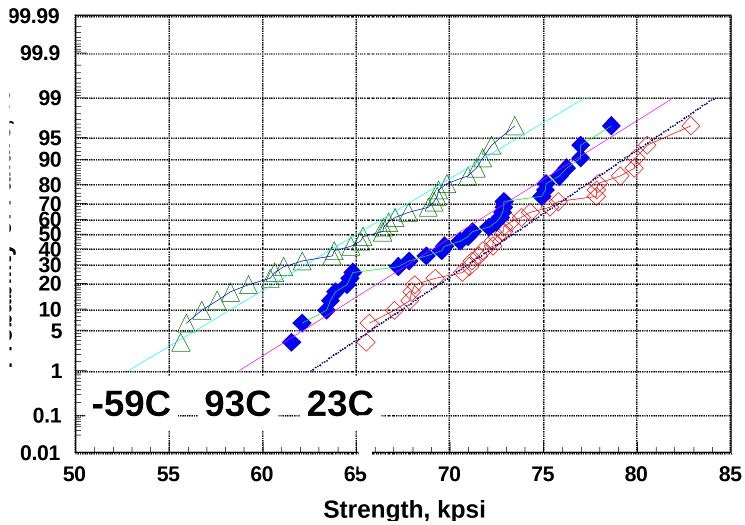

Figure 2: Cumulative probabilty plot.

- Confidence limits

$$\text{distribution of means} \quad s_m = \frac{s}{\sqrt{N}}$$

Since 95% of all measurements of a normally distributed population lie within 1.96 standard deviations from the mean, the ratio $\pm 1.96 s/\sqrt{N}$ is the range over which we can expect 95 out of 100 measurements of the mean to fall.

- Goodness of fit

$$\chi^2 = \sum \frac{(\text{expected} - \text{observed})^2}{\text{observed}}$$

$$= \sum_{i=1}^{N} \frac{(Np_i - n_i)^2}{n_i}$$

where N is the total number of specimens, n_i is the number of specimens actually failing in a strength increment $\Delta\sigma_{f,i}$ and p_i is the probability expected from the assumed distribution of a specimen having having a strength in that increment.

Lower Limit	Upper Limit	Observed Frequency	Expected Frequency	Chisquare
0	69.33	7	5.9	0.198
69.33	72.00	5	5.8	0.116
72.00	74.67	8	6.8	0.214
74.67	77.33	2	5.7	2.441
77.33	∞	8	5.7	0.909
				$\chi^2 = 3.878$

The number of degrees of freedom for this Chi-square test is 4; this is the number of increments less one, since we have the constraint that $n_1 + n_2 + n_3 + n_5 = 30$. From Table 3 in Appendix H, we read that $\alpha = 0.05$ for $\chi^2 = 9.488$, where α is the fraction of the χ^2 population with values of χ^2 greater than 9.488.

- The "B-allowable."

  The "B-allowable" strength is the stress level for which we have 95% confidence that 90% of all specimens will have at least that strength.

$$B = \overline{\sigma_f} - k_B\, s$$

where k_b is $n^{-1/2}$ times the 95th quantile of the "noncentral t-distribution;" this factor is tabulated, but can be approximated by the formula

$$k_b = 1.282 + \exp(0.958 - 0.520 \ln N + 3.19/N)$$

In the case of the previous 30-test example, k_B is computed to be 1.78, so this is less conservative than the $3s$ guide. The B-basis value is then

$$B = 73.28 - (1.78)(4.632) = 65.05$$

- The Weibull distribution

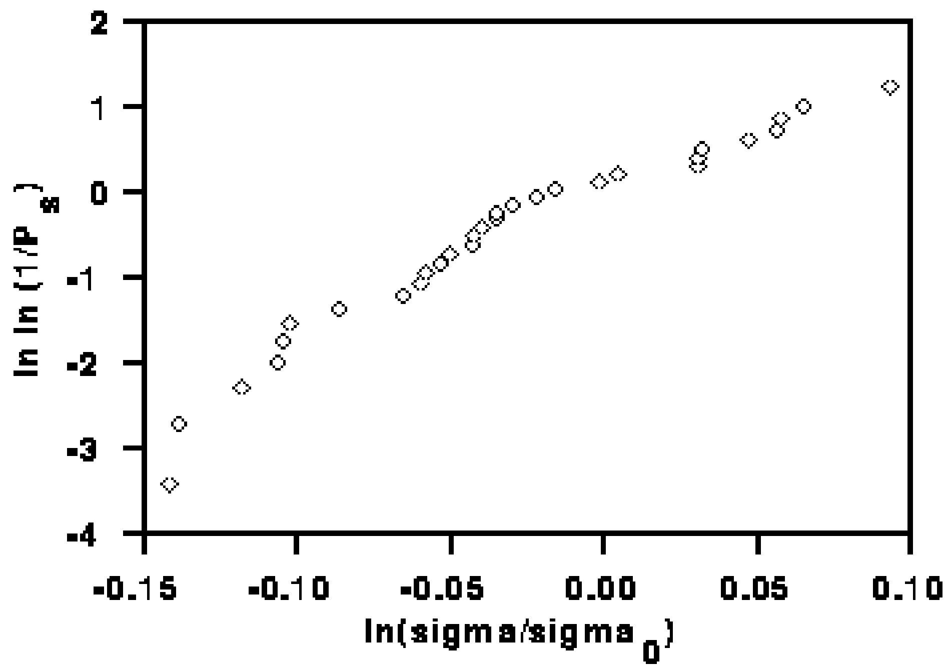

Figure 3: Weibull plot of strength data.

$$\ln P_s = -\left(\frac{\sigma}{\sigma_0}\right)^m$$

$$\ln(\ln P_s) = -m \ln\left(\frac{\sigma}{\sigma_0}\right)$$

Hence the double logarithm of the probability of exceeding a particular strength σ versus the logarithm of the strength should plot as a straight line with slope m.

The Weibull equation can be used to predict the magnitude of the size effect. If for instance we take a reference volume V_0 and express the volume of an arbitrary specimen as $V = nV_0$, then the probability of failure at volume V is found by multiplying $P_s(V)$ by itself n times:

$$P_s(V) = [P_s(V_0)]^n = [P_s(V_0)]^{V/V_0}$$

$$P_s(V) = \exp -\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m$$

Hence the probability of failure increases exponentially with the specimen volume.

- Remember Mark Twain's aphorism:

  There are lies, damned lies, and statistics.