MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 4 6.632 Electromagnetic Wave Theory
Spring Term 2003

Reading assignment: Section 3.3, 3.4 J. A. Kong, "*Electromagnetic Wave Theory*"

Problem P4.1

For a bianisotropic medium with the constitutive relation

$$\overline{D} = \overline{\overline{\epsilon}} \cdot \overline{E} + i\chi\overline{H}$$
$$\overline{B} = \mu\overline{H} - i\chi\overline{E}$$

with $\overline{\overline{\epsilon}}$ given by (E3.1.2.1) (Page 309), derive the wave vector along the $\hat{z}$- direction and discuss the polarization states of the characteristic waves in the medium.

Problem P4.2

Compare the phenomena of total reflection for $\theta > \theta_C$ and total transmission for $\theta = \theta_B$ at an isotropic dielectric interface.

(a) Total reflection occurs at a range of incident angles larger than the critical angle θ_C ; total transmission of TM waves occurs only at the Brewster angle θ_B .

(b) Total reflection occurs only when the incident medium is denser than the transmitted medium. The Brewster angle occurs for any two media.

(c) When an unpolarized wave is totally reflected, the reflected wave is still unpolarized. When the TM wave components of an unpolarized wave are totally transmitted, the reflected wave contains only TE waves.

Suppose a TM wave is incident at an angle θ such that $\theta = \theta_B > \theta_C$. Then the wave is totally transmitted and at the same time it is totally reflected. Explain what is happening.

Problem P4.3

The direction of propagation of a wave becomes ambiguous in a complex medium. From Poynting's theorem, we have learned that the energy flow of an electromagnetic field is governed by Poynting's vector, $\overline{S} = \overline{E} \times \overline{H}$. The Poynting's vector divided by the total electromagnetic energy is referred to as the *energy velocity*. The direction of the energy velocity is thus perpendicular to both $\overline{E}$ and $\overline{H}$. We have also learned that the direction of the phase velocity is along $\overline{k}$, which is perpendicular to both $\overline{D}$ and $\overline{B}$. In a bianisotropic medium, the directions of the energy velocity and the phase velocity $\overline{k}$ do not, in general, coincide.

The Poynting power-flow direction is characterized by the ray vector $\overline{s}$. We defined the magnitude of $\overline{s}$ by

$$\overline{s} \cdot \overline{k} = 1$$

where $\overline{s}$ is perpendicular to both $\overline{E}$ and $\overline{H}$:

$$\overline{s} \cdot \overline{E} = 0$$
$$\overline{s} \cdot \overline{H} = 0$$

The ray vector $\overline{s}$ has the dimension of length.

(a) Use the vector identity $\overline{s} \times (\overline{k} \times \overline{E}) = \overline{k}(\overline{s} \cdot \overline{E}) - (\overline{k} \cdot \overline{s})\overline{E}$ to show that

$$\overline{s} \times \overline{B} = -\frac{\overline{E}}{\omega}$$
$$\overline{s} \times \overline{D} = \frac{\overline{H}}{\omega}$$

(b) Define ray surfaces similar to the wave surfaces. Show that

$$s_x^2 + s_y^2 + s_z^2 = \frac{1}{\omega^2 \mu \epsilon}$$

for the ordinary wave and

$$s_x^2 + s_y^2 + \frac{\epsilon}{\epsilon_z} s_z^2 = \frac{1}{\omega^2 \mu \epsilon_z}$$

for the extraordinary wave. Plot the ray surfaces for the negative and the positive uniaxial media.

(c) Since $\overline{s}$ is along the direction of energy velocity, $\overline{s} \cdot \delta\overline{k} = 0$; namely the normal to the wave surface gives the direction of the corresponding ray vector. Prove that the normal to the ray surface gives the direction of the corresponding $\overline{k}$ vector.

Problem P4.4

The ionosphere extends from approximately 50 km above the earth to several earth radii (mean earth radius is about 6371 km) with the maximum in ionization density at about 300 km. For simplicity, assume that the ionosphere consists of a 40 km thick E layer with electron density $N = 10^{11}\ \mathrm{m}^{-3}$ below a 200 km thick F layer with $N = 6 \times 10^{11}\ \mathrm{m}^{-3}$.

(a) What are the plasma frequencies of the E and F layers ?

(b) Consider a plane wave of 10 MHz incident at an angle θ upon the ionosphere from below the E layer, what is the angle θ_t of the wave in the ionospheric E layer ?

(c) Let $\theta = 30°$, below what frequency will the wave be totally reflected by the E layer and below what frequency will it be totally reflected by the F layer ?