

Electromagnetics and Applications

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Preface

The initial development of electrical science and engineering a century ago occurred almost entirely within the domain of electromagnetics. Most electrical curricula remained polarized around that theme until the mid-twentieth century when signal, device, and computational subjects became dominant. Continued expansion of the field has currently relegated undergraduate electromagnetics to perhaps a one-semester subject even though electromagnetic technology has expanded substantially and is basic to most applications. To meet the increasing educational challenge of providing both breadth and depth in electromagnetics within a brief presentation, this text uses a more physics-based approach and novel methods of explaining certain phenomena. It introduces students to electrodynamics across the entire range from statics to dynamics, and from motors to circuits, communications, optical fibers, and lasers. For example, we currently cover approximately ninety percent of the text in a one-semester subject meeting with faculty four hours per week. The text could also support undergraduate offerings over two quarters or even two semesters, and could perhaps also be used at the entry graduate level.

The main objectives of the text are to: 1) convey those big ideas essential to understanding the electromagnetic aspects of modern electrical and computer systems, 2) expose students to enough examples to make the big ideas tangible and erase most naiveté about dominant applications, 3) provide computational experience with Maxwell's equations sufficient to treat the basic examples, 4) provide the understanding and skills prerequisite to follow-on subjects, and 5) reinforce prior exposure to physics, mathematics, and electrical systems so as to help integrate student learning, including problem solving and design methods.

The first two chapters are the core of the text. They review the basic physics of electromagnetics and electromechanics and introduce the Lorentz force law, Maxwell's equations, media, boundary conditions, static field solutions, uniform plane waves, and power and energy. Although the chapters are best read sequentially, the four topical areas that follow the core can be read in any sequence and include: 1) Chapters 3, 5, and 6, which treat RLC devices and circuits; electromagnetic forces on charges, conductors, and media; and motors, 2) Chapters 4, 7, and 8, which treat quasistatics, solutions to Laplace's equation, and TEM lines, including matching, resonators, and transients, 3) Sections 4.1–4.3 plus Chapter 9, which treat field relaxation, non-uniform plane waves, reflection, waveguides, and cavity resonators, and 4) Chapters 10 and 11, which treat radiation, wire and aperture antennas, and applications such as communications systems and radar. Two “capstone” chapters then follow: Chapter 12 introduces optical waveguides, laser amplifiers, laser oscillators, and other optical devices (Chapters 9 and 11 are prerequisites), and Chapter 13 reviews most wave phenomena in an acoustic context after Chapters 7, 9, and 10 have been covered. This organization permits use of the text in a wide variety of formats, including one- and two-semester options. Most prerequisites are reviewed briefly in the Appendix or within the text. Future versions will have home problems and more examples.

Special thanks are owed to the many MIT faculty who have taught this subject and its three merged predecessors while sharing their insights with the author over the past forty years. Without such collegial participation the scope and brevity of this text would not have been

possible. The sections on waves, optics, acoustics, resonators, and statics benefited particularly from interactions with Professors Kong and Haus, Ippen and Bers, Stevens and Peake, Smullin, and Haus and Zahn, respectively. Scott Bressler and Laura von Bosau have been particularly helpful in reducing the graphics and text to the printed page.

This is a preliminary version of the final text and therefore any comments on content or potential additions or corrections would be appreciated.

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