12.510 Introduction to Seismology
Spring 2008

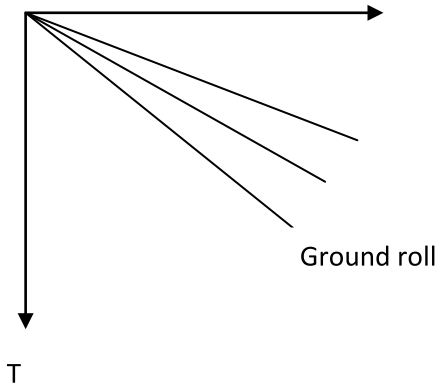
Continents: Quick review. Surface waves

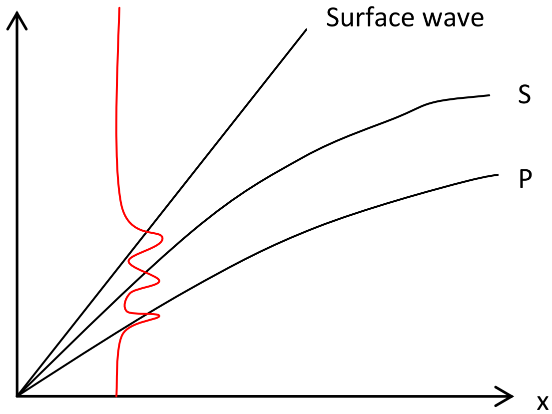
Ground roll-acoustic $\ddot{p} = V.\nabla.\left(\frac{1}{p}\nabla p\right)$. where p is the pressure

Love waves – SH $\ddot{u}_y = \mu.\nabla.\left(\frac{1}{\mathrm{p}}\nabla \mathrm{u_y}\right)$

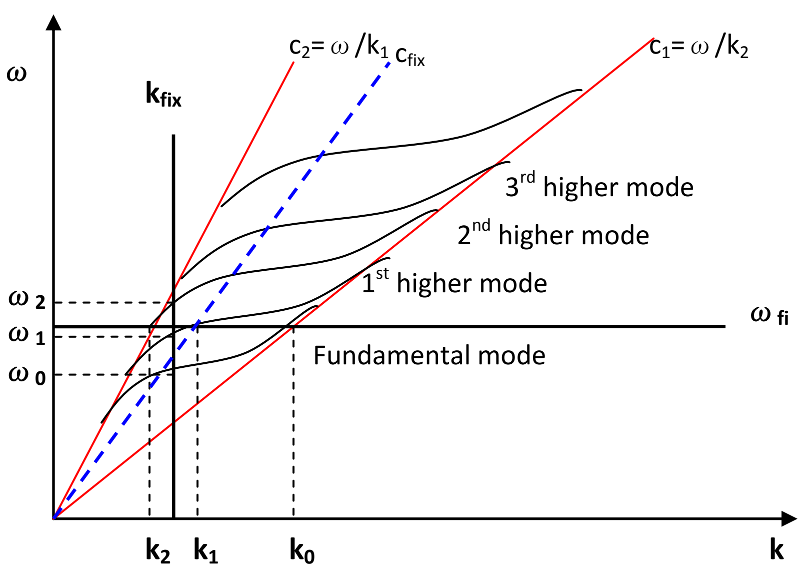
Rayleigh waves P-SV

Quick review (refer to April, 4, 2008 for details)

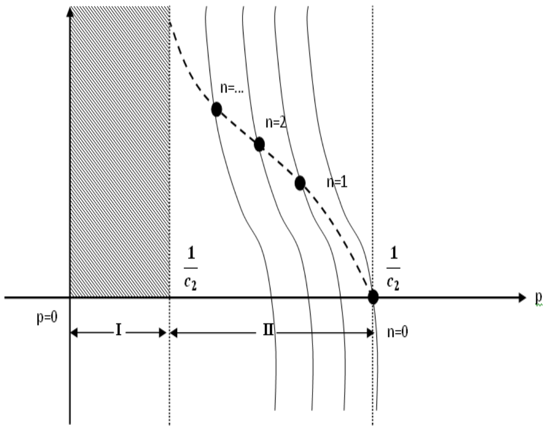




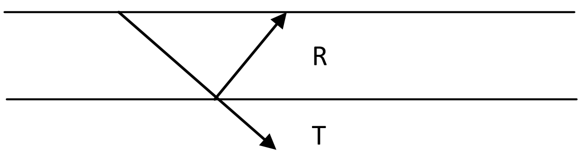
ω – k domain



Ground roll dispersion relationship

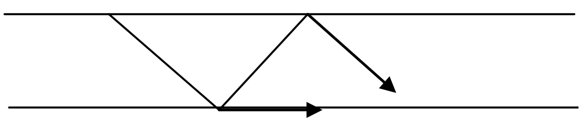


Pre-cretical $\mathbf{p} < \frac{\mathbf{1}}{\mathbf{c_2}}$

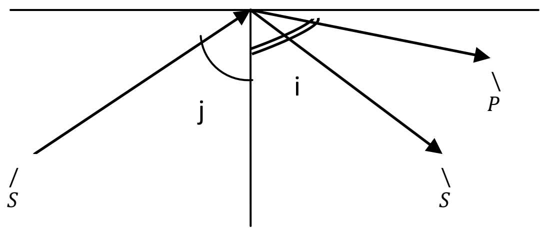


$$\eta_2 = \sqrt{\frac{1}{{c_2}^2} - p^2} \in \mathbb{R} \tag{1}$$

Post-critical $\mathbf{p} > \frac{\mathbf{1}}{\mathbf{c_2}}$

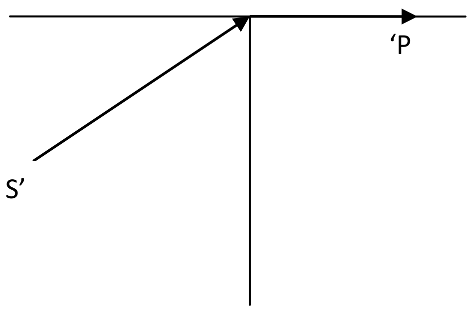


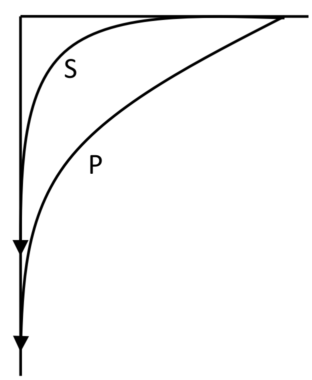
$$\eta_2 = \mathrm{i}\sqrt{p^2 - \frac{1}{{c_2}^2}} = \mathrm{i}\eta_2 \in \mathbb{C} \tag{2}$$



$$\frac{sin(j)}{\beta} = \frac{sin(i)}{\alpha} = \mathrm{p} \tag{3}$$

if β < α, there can be critical reflection and horizontal propagating p-wave. If j > j_c then there will be evanescence in the p-wave (p > 1/ α).





Both p-wave and s-wave horizontal propagation if $p > \frac{1}{\beta} > \frac{1}{\alpha} = \frac{1}{c}$. If a wave comes in with a 1/c that is larger than local 1/α and 1/β, the above will occur. This will also happen if the source emits a horizontal energy (rare).

$$P: \emptyset = Aexp(-\omega\eta_\alpha z)exp(i\omega(px-t)), \eta_\alpha = \sqrt{\frac{1}{\alpha^2} - p^2} \underbrace{=}_{p>\frac{1}{\alpha}} i\sqrt{p^2 - \frac{1}{\alpha^2}} = i\widehat{\eta_\alpha} = i\sqrt{\left(\frac{1}{c}\right)^2 - \left(\frac{1}{\alpha}\right)^2}; \text{ where } c = c_R = \frac{1}{p} \ (c_R\text{: phase velocity for Rayleigh waves}) \tag{4}$$

$$S: \psi = \beta exp\left(-\omega\eta_\beta z\right)exp(i\omega(px-t)), \eta_\beta = \sqrt{\frac{1}{\beta^2} - p^2} \underbrace{=}_{p>\frac{1}{\beta}} i\sqrt{p^2 - \frac{1}{\beta^2}} = i\widehat{\eta_\beta} = i\sqrt{\left(\frac{1}{c}\right)^2 - \left(\frac{1}{\beta}\right)^2} \tag{5}$$

We follow the same "Recipe" we used before:

1. Potentials
2. Boundary Conditions (Kinematic and dynamic)
3. Zoeppritz equatios.

Boundary conditions

$$u(x,t) = \nabla\emptyset + \nabla \times \psi \tag{6}$$

In this case

$$R_{\overset{/}{s}\overset{\backslash}{s}}, R_{\overset{/}{s}\overset{\backslash}{p}}, R_{\overset{/}{p}\overset{\backslash}{s}}, R_{\overset{/}{p}\overset{\backslash}{p}} \tag{7}$$

After some work we get:

$$\begin{gathered} A[(\lambda + 2\mu)\eta_\alpha^2 + \lambda p^2] + 2\mu p\eta_\beta = 0 \\ A(2p\eta_\alpha) + B\left(p^2 - \eta_\beta^2\right) = 0 \end{gathered} \tag{9}$$

Zoepprtiz

$$\begin{bmatrix} (\lambda + 2\mu)\eta_\alpha^2 + \lambda p^2 & 2\mu p\eta_\beta \\ 2p\eta_\alpha & p^2 - \eta_\beta^2 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{10}$$

Trivial solution is
$\boldsymbol{A = B = 0}$

Non-trivial solution leads to:

$$[(\lambda + 2\mu)\eta_\alpha^2 + \lambda p^2]\left[p^2 - \eta_\beta^2\right] - 2p\eta_\alpha(2\mu p\eta_\beta) = 0 \tag{11}$$

This expression; Rayleigh wave denominator (Rayleigh, 1887), can be written looking at wave speeds, but usually done numerically assuming a Poisson's medium (λ=μ) and $\alpha = \sqrt{3}\beta$. This scaling will help to simplify the above equation.

$$\begin{gathered} \boldsymbol{\eta_\alpha = \sqrt{\frac{1}{\alpha^2} - p^2} = \sqrt{\frac{1}{\alpha^2} - \left(\frac{1}{c}\right)^2}} \\ \boldsymbol{\eta_\beta = \sqrt{\frac{1}{\beta^2} - p^2} = \sqrt{\frac{1}{\beta^2} - \left(\frac{1}{c}\right)^2}} \end{gathered} \tag{12}$$

$$\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

$$\Rightarrow \quad \rho\alpha^2 = \lambda + 2\mu$$

$$\beta = \sqrt{\frac{\mu}{\rho}}$$

$$\Rightarrow \quad \rho\beta^2 = \mu$$

$$\left[\alpha^2\left(\frac{\eta_\alpha^2}{\rho^2} + 1\right) - 2\beta^2\right]\left(1 - \frac{\eta_\beta^2}{\rho^2}\right) - \left(\frac{4\beta^2\eta_\alpha\,\eta_\beta}{\rho^2}\right) = 0$$

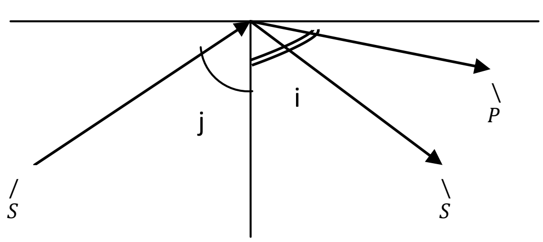
$$\Rightarrow \quad \frac{\beta}{\alpha}$$

Assumptions
Poisson medium: $\lambda = \mu$
Poisson ratio: $V = \frac{\lambda}{2(\lambda+\mu)} = 0.25$
$\alpha = \sqrt{3}\beta$

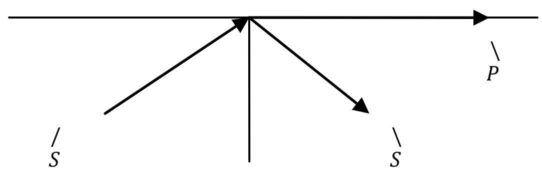
$$\boxed{\left(\frac{c_R^2}{\beta^2}\right)^3 - 8\left(\frac{c_R^2}{\beta^2}\right)^2 + \frac{56}{3}\left(\frac{c_R^2}{\beta^2}\right) - \frac{32}{3} = 0}$$

Three solutions

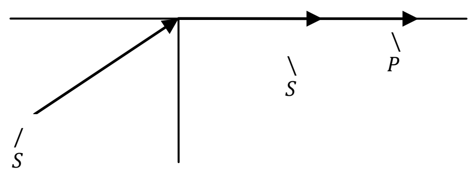
1. $\left(\frac{c}{\beta}\right)^2 = 4 \Rightarrow c = 2\beta, c > \beta, p = \frac{1}{c} < \frac{1}{p}, p < \frac{1}{\alpha}$



2. $\left(\frac{c}{\beta}\right)^2 = 2 + \frac{2}{3}\sqrt{3}, \frac{1}{\alpha} < p < \frac{1}{\beta}, c^2 = (2 + \frac{2}{3}\sqrt{3})\beta^2$



3. $\left(\frac{c}{\beta}\right)^2 = (2 - \frac{2}{3}\sqrt{3}), \frac{1}{\alpha} < p < \frac{1}{\beta}, c = \sqrt{2 + \frac{2}{3}\sqrt{3}}, c < \beta, p = \frac{1}{c} > \frac{1}{\beta} > \frac{1}{\alpha}, (\eta_\alpha = i\widehat{\eta_\alpha}, \eta_\beta = i\widehat{\eta_\beta})$



Note: $c_R \sim 0.92\beta$ which makes it about 10% slower than the shear wave velocity. This explains why the Rayleigh wave is slower than the love wave. The Love wave, at low frequency → c_2 and at high frequencies Love wave (at it's slowest) is c_1…Rayleigh wave is about 90% of the Love wave at the most.

exponential decay with depth

$$\emptyset = A exp(-\omega\widehat{\eta_\alpha}z)exp(i\omega(px - t))$$
$$\psi = \beta exp(-\omega\widehat{\eta_\beta}z)exp(i\omega(px - t)) \quad (13)$$

$$u = \nabla\emptyset + \nabla \times \psi$$

$$u_x = \frac{\partial\emptyset}{\partial x} - \frac{\partial\psi}{\partial z}; \; u_z = \frac{\partial\emptyset}{\partial z} - \frac{\partial\psi}{\partial x} \quad (14)$$

$$u_x \sim A\sin(kx - \omega t)\ldots$$

$$u_z \sim A\cos(kx - \omega t)\ldots$$

u_x, u_z are $\frac{\pi}{2}$ out of phase

Updated by: Sami Alsaadan
Sources: April 13, 2008 by Patricia Gregg. April 8, 2008 lecture.
"An Introduction to Seismology, Earthquakes, And Earth Structure" by Stein &Wysession (2007).